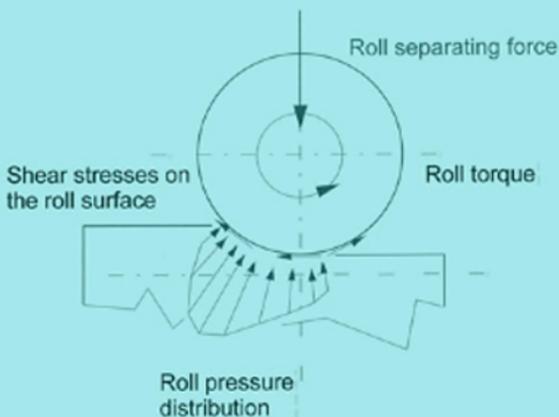
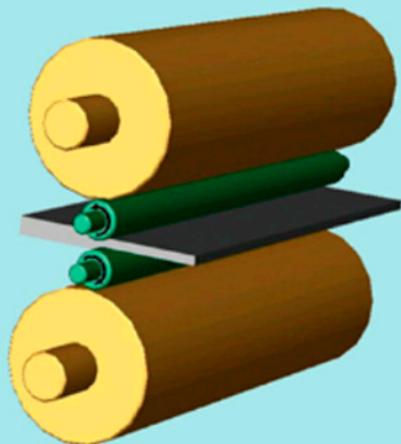


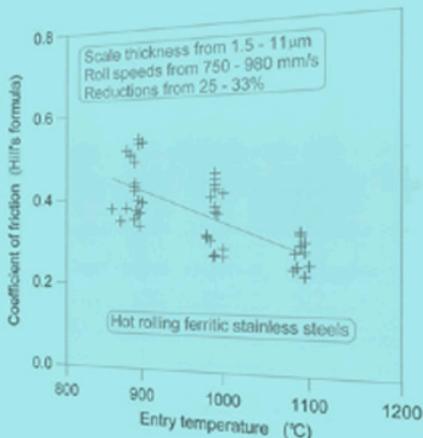


$$\frac{d}{dx} \left[ h \left( p - 2k \mp \tau \frac{dy}{dx} \right) \right] = 2 \left( p \frac{dy}{dx} \pm \tau \right)$$



# Primer on Flat Rolling

A monograph for those who need to understand the basics of the flat rolling process



John G. Lenard

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# **Primer on Flat Rolling**

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# Primer on Flat Rolling

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By

JOHN G. LENARD



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I dedicate this book to my wife, Harriet and my daughter, Patti.  
Their active support, encouragement and love made  
the writing possible and easy.

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# CONTENTS

<i>Preface</i>	xv
<i>List of Symbols</i>	xviii
<i>Advice for Instructors</i>	xxi
<b>1 Introduction</b>	<b>1</b>
Abstract	1
1.1 The Flat Rolling Process	1
1.1.1 <i>Hot, cold and warm rolling</i>	2
1.2 The Hot Rolling Process	2
1.2.1 <i>Reheating furnace</i>	3
1.2.2 <i>Rough rolling</i>	4
1.2.3 <i>Coil box</i>	4
1.2.4 <i>Finish rolling</i>	6
1.2.5 <i>Cooling</i>	7
1.2.6 <i>Coiling</i>	7
1.2.7 <i>The hot strip mill</i>	8
1.2.8 <i>The Steckel mill</i>	8
1.3 Continuous Casting	9
1.4 Mini-Mills	10
1.5 The Cold Rolling Process	11
1.5.1 <i>Cold rolling mill configurations</i>	11
1.6 The Warm-Rolling Process	14
1.7 New Equipment	15
1.8 Further Reading	15
1.9 Conclusions	16
<b>2 Flat Rolling – A General Discussion</b>	<b>17</b>
Abstract	17
2.1 The Flat Rolling Process	17
2.1.1 <i>Hot, cold and warm rolling</i>	19
2.1.2 <i>Mathematical modelling</i>	19
2.1.3 <i>The independent and dependent variables</i>	20
2.2 The Physical Events Before, During and After the Pass	22
2.2.1 <i>Some assumptions and simplifications</i>	28
2.2.1.1 <i>Plane-strain flow</i>	28
2.2.1.2 <i>Homogeneous compression</i>	28
2.3 The Metallurgical Events Before and After the Rolling Process	30

2.4	Limitations of the Flat Rolling Process	32
2.4.1	<i>The minimum rollable thickness</i>	33
2.4.2	<i>Alligatoring and edge-cracking</i>	34
2.5	Conclusions	35
<b>3</b>	<b>Mathematical and Physical Modelling of the Flat Rolling Process</b>	<b>36</b>
	Abstract	36
3.1	A Discussion of Mathematical Modelling	36
3.2	A Simple Model	41
3.3	One-dimensional Models	47
3.3.1	<i>The Classical Orowan model</i>	47
3.3.2	<i>Sims' model</i>	50
3.3.3	<i>Bland and Ford's model</i>	51
3.4	Refinements of the Orowan Model	52
3.4.1	<i>The deformation of the work roll</i>	55
3.5	The Effect of the Inertia Force	57
3.5.1	<i>The equations of motion</i>	58
3.5.2	<i>A numerical approach</i>	58
3.6	The Predictive Ability of the Mathematical Models	59
3.7	The Friction Factor in the Flat Rolling Process	61
3.7.1	<i>The mathematical model</i>	63
3.7.2	<i>Calculations using the model</i>	65
3.7.2.1	<i>Cold rolling of steel</i>	65
3.7.2.2	<i>Distribution of the roll pressure at the contact</i>	68
3.8	The Use of ANN	70
3.8.1	<i>Structure and terminology</i>	70
3.8.2	<i>Interconnection</i>	71
3.8.3	<i>Propagation of information</i>	71
3.8.4	<i>Functions of a node</i>	71
3.8.5	<i>Threshold function</i>	71
3.8.6	<i>Learning</i>	71
3.8.7	<i>Characteristics of neural networks</i>	72
3.8.8	<i>Back-propagation neural networks</i>	72
3.8.9	<i>General Delta Rule</i>	73
3.8.10	<i>The learning algorithm</i>	73
3.8.11	<i>Drawbacks of B-P networks</i>	73
3.8.12	<i>Application of neural networks to predict the roll forces in cold rolling of a low carbon steel</i>	73
3.9	Extremum Principles	74
3.9.1	<i>The upper bound theorem</i>	75
3.10	Comparison of the Predicted Powers	78
3.11	The Development of the Metallurgical Attributes of the Rolled Strip	78
3.11.1	<i>Thermal–mechanical treatment</i>	80
3.11.1.1	<i>Controlled rolling of C–Mn steels</i>	82

3.11.1.2	<i>Dynamic and metadynamic recrystallization-controlled rolling</i>	82
3.11.1.3	<i>Effects of recrystallization type on the grain size</i>	83
3.11.1.4	<i>Controversies regarding the type of recrystallization in strip rolling</i>	83
3.11.2	<i>Conventional microstructure evolution models</i>	84
3.11.2.1	<i>Static changes of the microstructure</i>	84
3.11.2.2	<i>Dynamic softening</i>	88
3.11.2.3	<i>Metadynamic recrystallization</i>	89
3.11.2.4	<i>Grain growth</i>	89
3.11.3	<i>Properties at room temperatures</i>	90
3.11.3.1	<i>Ferrite grain size</i>	90
3.11.3.2	<i>Lower yield stress</i>	90
3.11.3.3	<i>Tensile strength</i>	90
3.11.4	<i>Physical simulation</i>	91
3.12	<i>Miscellaneous Parameters and Relationships in the Flat Rolling Process</i>	91
3.12.1	<i>The forward slip</i>	91
3.12.2	<i>Mill stretch</i>	92
3.12.3	<i>Roll bending</i>	92
3.12.4	<i>Cumulative strain hardening</i>	93
3.12.5	<i>The lever arm</i>	94
3.13	<i>How a Mathematical Model should be Used</i>	96
3.13.1	<i>Establish the magnitude of the coefficient of friction</i>	96
3.13.2	<i>Establish the metal's resistance to deformation</i>	96
3.14	<i>Conclusions</i>	97

**4 Material Attributes 99**

Abstract	99
4.1 Introduction	99
4.2 Recently Developed Steels	100
4.2.1 <i>Very low carbon steels</i>	101
4.2.2 <i>Interstitial free (IF) steels</i>	101
4.2.3 <i>Bake-hardening (BH) steels</i>	102
4.2.4 <i>TRIP steel</i>	102
4.2.5 <i>High strength low alloy (HSLA) steels</i>	103
4.2.6 <i>Dual-phase (DP) steels</i>	105
4.3 Steel and Aluminum	106
4.4 The Independent Variables	108
4.5 Traditional Testing Techniques	109
4.5.1 <i>Tension tests</i>	109
4.5.2 <i>Compression testing</i>	111
4.5.3 <i>Torsion testing</i>	113
4.6 Potential Problems Encountered During the Testing Process	114
4.6.1 <i>Friction control</i>	114
4.6.2 <i>Temperature control</i>	116

4.6.2.1	<i>Isothermal conditions</i>	116
4.6.2.2	<i>Monitoring the temperature</i>	117
4.7	The Shape of Stress–Strain Curves	118
4.7.1	<i>Low temperatures</i>	118
4.7.2	<i>High temperatures</i>	119
4.8	Mathematical Representation of Stress–Strain Data	121
4.8.1	<i>Material models: stress–strain relations</i>	123
4.8.1.1	<i>Relations for cold rolling</i>	123
4.8.1.2	<i>Relations for use in hot rolling</i>	123
4.9	Choosing a Stress–Strain Relation for Use in Modelling the Rolling Process	129
4.10	Conclusions	129
<b>5</b>	<b>Tribology</b>	<b>130</b>
	Abstract	130
5.1	Tribology – A General Discussion	130
5.2	Friction	132
5.2.1	<i>Real surfaces</i>	132
5.2.2	<i>The areas of contact</i>	132
5.2.2.1	<i>The relationship of the apparent and the true areas of contact</i>	135
5.2.3	<i>Definitions of frictional resistance</i>	139
5.2.4	<i>The mechanisms of friction</i>	141
5.3	Determining the Coefficient of Friction or the Friction Factor	142
5.3.1	<i>Experimental methods</i>	142
5.3.1.1	<i>The embedded pin – transducer technique</i>	142
5.3.1.2	<i>The refusal technique</i>	144
5.3.1.3	<i>The ring compression test</i>	144
5.3.2	<i>Semi-analytical methods</i>	146
5.3.2.1	<i>Forward slip – coefficient of friction relations</i>	146
5.3.2.2	<i>Empirical equations – cold rolling</i>	150
5.3.2.3	<i>The study of Tabary et al. (1994)</i>	152
5.3.2.4	<i>Empirical equations and experimental data – hot rolling</i>	153
5.3.2.5	<i>Inverse calculations</i>	157
5.3.2.6	<i>Negative forward slip</i>	158
5.3.2.7	<i>The correlation of the coefficient of friction, determined in the laboratory and in industry</i>	159
5.4	Lubrication	160
5.4.1	<i>The lubricant</i>	160
5.4.1.1	<i>The viscosity</i>	160
5.4.1.2	<i>The viscosity–pressure relationship</i>	161
5.4.1.3	<i>The viscosity–temperature relationship</i>	163
5.4.1.4	<i>The combined effect of temperature and pressure on viscosity</i>	164

5.4.2	<i>The lubrication regimes</i>	164
5.4.3	<i>A well-lubricated contact in flat rolling</i>	166
5.4.4	<i>Neat oils or emulsions?</i>	167
5.4.4.1	<i>Roll force and roll torque</i>	167
5.4.4.2	<i>The coefficient of friction</i>	170
5.4.5	<i>Oil-in-water emulsions</i>	171
5.4.5.1	<i>Behaviour of the droplets</i>	172
5.4.5.2	<i>Entrainment of the emulsion</i>	173
5.4.5.3	<i>The emulsion in the contact zone</i>	174
5.4.6	<i>A physical model of the contact of the roll and the strip</i>	174
5.4.7	<i>The thickness of the oil film</i>	175
5.4.7.1	<i>Measurement of the thickness of the oil film</i>	176
5.4.7.2	<i>Calculation of the oil film thickness</i>	178
5.5	<i>Dependence of the Coefficient of Friction or the Roll Separating Force on the Independent Variables</i>	179
5.5.1	<i>The dependence of coefficient on reduction</i>	180
5.5.2	<i>The dependence of coefficient on speed</i>	181
5.5.3	<i>The dependence of coefficient on the surface roughness of the roll</i>	183
5.5.4	<i>The dependence of the roll separating force on the lubricant's viscosity</i>	184
5.5.5	<i>The dependence of the coefficient of friction on temperature</i>	186
5.5.5.1	<i>The layer of scale</i>	186
5.5.5.2	<i>The effect of the scale thickness on friction</i>	189
5.6	<i>Heat Transfer</i>	190
5.6.1	<i>Estimating the heat transfer coefficient on a laboratory rolling mill</i>	192
5.6.2	<i>Measuring the surface temperature of the roll</i>	194
5.6.3	<i>Hot rolling in industry – the heat transfer coefficient on production mills</i>	195
5.7	<i>Roll Wear</i>	195
5.8	<i>Conclusions</i>	198
5.8.1	<i>Heat transfer coefficient</i>	201
5.8.2	<i>The coefficient of friction</i>	202
5.8.2.1	<i>Cold rolling</i>	202
5.8.2.2	<i>Hot rolling</i>	202
5.8.3	<i>Roll wear</i>	202
5.8.4	<i>What is still missing</i>	203
<b>6</b>	<b>Applications and Sensitivity Studies</b>	<b>204</b>
	<i>Abstract</i>	204
6.1	<i>The Sensitivity of the Predictions of the Flat Rolling Models</i>	204
6.1.1	<i>The sensitivity of the roll separating force and the roll torque to the coefficient of friction and the reduction</i>	205

6.1.2	<i>The sensitivity of the roll separating force and the roll torque to the strain-hardening co-efficient</i>	207
6.1.3	<i>The dependence of the roll separating force and the roll torque on the entry thickness</i>	208
6.2	A Comparison of the Predictions of Power, Required for Plastic Deformation of the Strip	209
6.3	The Roll Pressure Distribution	210
6.4	The Statically Recrystallized Grain Size	211
6.5	The Critical Strain	213
6.6	The Hot Strength of Steels – Shida’s Equations	214
6.6.1	<i>The shape of the stress–strain curve, as predicted by Shida</i>	214
6.7	Conclusions	216
<b>7</b>	<b>Temper Rolling</b>	<b>217</b>
	Abstract	217
7.1	The Temper Rolling Process	217
7.2	The Mechanism of Plastic Yielding	218
7.3	The Effects of Temper Rolling	219
7.3.1	<i>Yield strength variation</i>	219
7.4	Mathematical Models of the Temper Rolling Process	220
7.4.1	<i>The Fleck and Johnson models</i>	220
7.4.2	<i>Roberts’ model</i>	221
7.4.3	<i>The model of Fuchshumer and Schlacher (2000)</i>	223
7.4.4	<i>The Gratacos and Onno model (1994)</i>	224
7.4.5	<i>The model of Domanti et al. (1994)</i>	224
7.4.6	<i>The Chandra and Dixit model (2004)</i>	225
7.4.7	<i>The models of Wiklund (1996a, 1996b, 1999, 2002)</i>	225
7.4.8	<i>The model of Liu and Lee (2001)</i>	226
7.4.9	<i>The studies of Sutcliffe and Rayner (1998)</i>	227
7.4.10	<i>The model of Pawelski (2000)</i>	228
7.5	Comments from Industry	229
7.6	Conclusions	229
<b>8</b>	<b>Severe Plastic Deformation – Accumulative Roll Bonding</b>	<b>230</b>
	Abstract	230
8.1	Introduction	230
8.2	Manufacturing Methods of Severe Plastic Deformation (SPD)	231
8.2.1	<i>High-pressure torsion</i>	231
8.2.2	<i>Equal channel angular pressing (ECAP)</i>	231
8.2.3	<i>Cyclic extrusion-compression</i>	232
8.2.4	<i>Multiple forging</i>	232
8.2.5	<i>Continuous confined strip shearing</i>	233
8.2.6	<i>Repetitive corrugation and straightening (RCS)</i>	233
8.2.7	<i>Accumulative roll-bonding (ARB)</i>	233

8.3	A Set of Experiments	236
8.3.1	<i>Material</i>	236
8.3.2	<i>Preparation and procedure</i>	236
8.3.3	<i>Equipment</i>	237
8.4	Results and Discussion	237
8.4.1	<i>Process parameters</i>	237
8.4.2	<i>Mechanical attributes at room temperature</i>	239
8.4.2.1	<i>Hardness</i>	239
8.4.2.2	<i>Yield, tensile strength and ductility</i>	239
8.4.2.3	<i>The bending strength</i>	240
8.4.2.4	<i>The cross-section of the roll-bonded strips</i>	240
8.4.2.5	<i>The strength of the bond</i>	241
8.5	The Phenomena Affecting the Bonds	242
8.5.1	<i>Cracking of the edges</i>	243
8.6	A Potential Industrial Application: Tailored Blanks	245
8.7	A Combination of ECAP and ARB	245
8.7.1	<i>The ECAP process</i>	246
8.7.2	<i>The rolling process</i>	247
8.7.3	<i>The microstructure after ECAP and the rolling passes</i>	248
8.8	Conclusions	251
<b>9</b>	<b>Roll Bonding</b>	<b>252</b>
	Abstract	252
9.1	Introduction	252
9.2	Material, Equipment, Sample Preparation, Parameters	254
9.2.1	<i>Material</i>	254
9.2.2	<i>Equipment</i>	255
9.2.3	<i>Sample preparation</i>	255
9.3	Parameters	255
9.4	Testing of the Shear Strength of the Bond	256
9.5	Results and Discussion	256
9.5.1	<i>The roll force and the torque</i>	256
9.5.2	<i>The shear strength of the bond</i>	256
9.5.2.1	<i>The effect of the speed of rolling</i>	256
9.5.2.2	<i>The effect of the normal pressure</i>	258
9.5.2.3	<i>The effect of the entry temperature – warm bonding</i>	259
9.5.2.4	<i>The effect of the entry temperature – cold bonding</i>	260
9.6	Examination of the Interface	261
9.6.1	<i>Warm bonding</i>	261
9.6.2	<i>Cold bonding</i>	263
9.6.3	<i>Side view of the bond</i>	263
9.7	The Phenomenon of Bonding	264
9.8	Conclusions	267

<b>10 Flexible Rolling</b>	<b>268</b>
Abstract	268
10.1 Introduction	268
10.2 Material, Equipment, Procedure, Sample Preparation	271
10.2.1 Material	271
10.2.2 Equipment	272
10.2.3 Procedure	272
10.2.4 Sample Preparation	272
10.3 Results and Discussion	272
10.3.1 Roll separating forces and the roll gap	272
10.3.1.1 AISI 1030 steel, cold drawn	272
10.3.1.2 AISI 1008 steel, cold drawn	274
10.3.1.3 Al 6111 aluminum alloy	275
10.4 Predictions of a Simple Model	276
10.5 Strain at Fracture	278
10.6 Conclusions	280
<b>11 Problems and Solutions</b>	<b>281</b>
Abstract	281
Part 1: Problems	281
Part 2: Solutions	302
<i>References</i>	313
<i>Author Index</i>	334
<i>Subject Index</i>	340

## PREFACE

I have been dealing with problems of the flat rolling process for the last 30 years. This included mathematical modelling, experimentation, consulting, publishing in technical journals, presenting my research at conferences and in industry, as well as lecturing on the topic at levels, appropriate for second and third year undergraduate students, graduate students and practicing engineers and technologists of aluminum and steel companies. The present book is a compilation of my experience, prepared for use by practitioners who work with metal rolling and who want to know about the “why”-s, the “what”-s and the interdependence of the material and process parameters of the rolling process. The book may also be useful for graduate students, researching flat rolling.

My interest in the process began while I spent a year at Stelco Research as an NSERC Senior Industrial Fellow, shortly after starting my academic career. I became aware of the tremendous complexity underlying the seemingly very simple process of metal rolling. I realized that while the process of flat rolling – that of two cylinders rotating in opposite directions and reducing the thickness of a strip as it passes between them – has not changed for centuries, its current sophistication places it at the top of the “high tech” activities. On return to academia, and as soon as research funds allowed, I designed and built a simple two-high experimental rolling mill and instrumented it to measure the important variables. The mill has been in use ever since to roll various metals – mostly aluminum and steel alloys – under a large variety of conditions. These conditions included dry and lubricated passes, use of neat oils and emulsions, high, low and intermediate temperatures, heated and non-heated rolls, speeds and reductions as high and low as the mill allowed. During these experiments, my students and I used smooth and rough roll surfaces, prepared by grinding or sand blasting.

In each of the tests, the roll separating forces, the roll torques, the entry and exit thickness, the rolling speed, the forward slip, the entry and exit temperatures of the strip, the roll’s surface temperature, the amount of the lubricant, the flow rate and the temperature of the emulsion, the droplet size in the emulsion, the change of the width and the reduction of the strips were measured.

In addition to the experiments performed by myself, by academic visitors from China, Egypt, Germany, Hungary, India, Israel, Japan, Poland and South Korea, and by my graduate students, twice each year my undergraduate

classes, typically 80–100 students strong, performed flat rolling tests, providing me with a very respectable collection of data.

Mathematical modelling of the process proceeded parallel to the experimental studies. The attention was on establishing the predictive abilities of the available models of the flat rolling process. The assumptions made in the derivation of the traditional 1D models were critically examined and were improved on by developing an advanced 1D model which makes use of as few arbitrary assumptions as possible. The use of finite-element models was also explored, in co-operation with Prof. Pietrzyk (University of Mining and Metallurgy, Krakow, Poland) and his colleagues and students.

During my academic career, I offered, once or twice a year, a specialist course on rolling, designed for technologists and engineers who work in the metal rolling industry. The educational level of the audience varied broadly, from those who completed high school to those with doctoral degrees. Each year I found two unchanging phenomena. The first was the shaky background my listeners possessed, essentially regardless of their education. When asked about the difference between engineering strains and true strains, about the difference between the plane-stress and the plane-strain conditions, the difference between static and dynamic recrystallization, and so on, the large majority of them betrayed serious ignorance. The second was the lack of a textbook that includes all I needed to develop the ideas in the course. The present book, resulting from the notes I used in these courses, attempts to compile, present and explain the disparate components, needed for a clear understanding of the topic.

The book contains 11 chapters. The first 10 of these deal with various aspects of the flat rolling process and the 11th presents a set of assignments and incomplete solutions, formulated to test the understanding of the reader of the material presented. Each chapter ends with a set of Conclusions.

The flat rolling process is defined in Chapter 1, the Introduction. The objectives are to give a very brief overview of the process. Details of the hot rolling process, using hot strip mills, are given. Continuous casting is described. The cold rolling process and cold mill configurations are presented next.

A general discussion of the rolling process is presented in Chapter 2. The components of a metal rolling system are defined. Reference is made to the rolling mill, designed by Leonardo da Vinci and the scale-model, built following his drawings. A description of the physical and the metallurgical events during the process is given, including the events as the strip to be rolled is ready to enter the roll gap, as it is partially reduced and as the process becomes one of steady-state. The independent variables of the system – the mill, the rolled metal and their interface – are listed. The minimum value of the coefficient of friction, necessary to commence the rolling process is given. Some of the simplifying assumptions that are usually made in mathematical models of the process of flat rolling are critically discussed: these include the idea of “plane-strain plastic flow” and “homogeneous compression of the

strip". Microstructures of a fully recrystallized Nb steel, an AISI 1008 steel and a cold-rolled low carbon steel are presented.

Mathematical modelling of the rolling process is the topic of Chapter 3. Traditional and more advanced models are discussed in terms of their capabilities as far as their predictions are concerned. Models for both mechanical and metallurgical events are included. The chapter ends with the identification of three parameters, necessary for efficient, accurate and consistent modelling: the coefficients of heat transfer and friction and the resistance of the material to deformation.

Chapters 4 and 5 treat these in turn; material behaviour and tribology, respectively. In both, the emphasis is on how the concepts are to be used when combined with the models, presented in the previous chapter.

The objectives in preparing Chapter 6 are somewhat different. The chapter is entitled "Sensitivity studies" and in spite of some examination of the sensitivity of the predictions in previous chapters, some more calculations and applications are added.

Temper rolling is considered in Chapter 7. The differences between the usual flat rolling process and temper rolling are pointed out. Several mathematical models are given and the assumptions made in their development are discussed. The components that should make up a complete model of the process are listed.

The tenor of the book changes at that point. In each of Chapters 8, 9 and 10 – accumulative roll-bonding, cold-roll bonding and flexible rolling, respectively – a review of the literature is followed by the detailed descriptions of experimental work.

Chapter 11 contains two sections. In the first, problems are listed, for each of the chapters. Some of these require the direct application of the expressions and the formulas presented in the book. Some answers require Internet searches. Some require development of computer programs. Some are suggested topics for seminars or class discussions. In the second part, the solutions are given. Again, this is done in a variety of ways: in some cases detailed solutions are given while in some others, only the numerical answers are indicated. As well, in some instances, only a set of hints and recommended approaches are suggested.

I would like to acknowledge the contributions of my undergraduate and graduate students without whom my research would not have progressed. Also, I would like to thank the visiting scientists with whom co-operation was always most enjoyable.

## LIST OF SYMBOLS

$Ar_1, Ar_3$	Austenite to ferrite transformation; stop and start temperatures, respectively
$A, A_r$	Apparent and true areas of contact, respectively
$B$	Material constant
$C$	Material constant, indicating strain-rate hardening; also half the distance between the roll centres in eq. 3.29
$[C], C_{eq}$	Carbon content, %; carbon equivalent
$D$	Diameter of the work roll
$D_{\alpha}, D_{\gamma}, D_r$	Ferrite and austenite grain size; austenite grain size after recrystallization, respectively
$D_{DRX}$	Austenite grain size after dynamic recrystallization
$D_{MD}$	Austenite grain size after metadynamic recrystallization
$E, E'$	Elastic modulus; composite elastic modulus, respectively
$F_1$	Inertia force
$H$	Hardness
$J^*$	Externally supplied power
$J_2$	Second invariant of the stress deviator tensor
$K$	Material constant, strength coefficient
$K_y$	Grain boundary unlocking term
$L$	The contact length
$M$	Roll torque for both rolls per unit width
$N$	Roll rpm
$P, P_{total}$	Power required for plastic deformation and the total power needed to drive the rolling mill, respectively
$P_n$	Friction losses in four roll-neck bearings
$P_r$	The roll separating force per unit width
$Q_p$	The pressure intensification factor; a multiplier, accounting for the shape factor and the coefficient of friction
$Q_{RX}$	Activation energy for recrystallization
$R', R$	The radius of the flattened roll (by Hitchcock's equation) and the original radius of the roll, respectively
$S, S_{\Gamma}, S_i$	Surface; also mill stiffness in eq. 3.94; surfaces in the upper bound theorem, eq. 3.61, respectively
$S_{ij}$	Components of the stress deviator tensor
$S_f, S_b$	The forward and the backward slip
$T, T_0$	Temperature; roll temperature some distance below the surface

$\Delta T_{\text{gain}}, \Delta T_{\text{loss}}$	Temperature rise and loss of the strip in the pass, respectively
$T_{\text{strip}}, T_{\text{roll}}$	Temperature of the strip and the roll, respectively
$T_{\text{NRX}}$	Temperature above which recrystallization will occur
$V$	Volume
$\dot{W}$	The flattening rate
$X, X_{\text{DRX}}$	The recrystallized volume fraction; dynamically recrystallized volume fraction, respectively
$X_{\text{MD}}$	Metadynamically recrystallized volume fraction
$Y$	Material constant
$Z$	Zener-Hollomon parameter
$a$	Acceleration
$a, b$	Constants
$c_p$	Specific heat of the rolled strip
$d$	Diameter of the roll-neck bearing
$f_t, f_n$	Friction force, and the normal force, respectively
$h$	The thickness of the strip
$h_{\text{entry}}$	The thickness of the strip at the entry
$h_{\text{exit}}$	The thickness of the strip at the exit
$h_{\text{ave}}$	The average of the entry and the exit thickness
$h_{\text{min}}$	The minimum rollable thickness
$h_{\text{np}}$	The thickness of the strip at the neutral point
$h_{\text{film,ave}}, h_s$	Average oil film thickness; smooth oil film thickness
$k$	Yield strength of the material in pure shear
$l_a$	The lever arm
$m, m_{\text{ave}}$	Strain rate hardening exponent; also mass; average friction factor, respectively
$n, n_1$	Strain hardening exponent
$p, p_{\text{ave}}$	Roll pressure; average roll pressure
$p, q, r, s$	Material constants
$p_s$	Shakedown pressure
$r$	Reduction
$t$	time
$v$	Speed of the rolled strip; also Poisson's ratio in equation 3.5
$v_r$	Roll surface speed
$w$	Width of the strip
$x$	Distance along the direction of rolling, measured from the line connecting the roll centres
$x_n$	Location of the neutral point
$\alpha, \bar{\alpha}$	Heat transfer coefficient; parameter in Hatta's equation
$\beta^*$	Correlation distance
$\gamma$	Austenite
$\delta_{ij}$	Kronecker's delta
$\varepsilon_{0.5X}$	Strain for 50% recrystallization